

On the orthogonal component of BSDEs in a Markovian setting

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Abstract - In this Note we consider a quadratic backward stochastic differential equation (BSDE) driven by a continuous martingale M and whose generator is a deterministic function. We prove (in Theorem 2.1) that if M is a strong homogeneous Markov process and if the BSDE has the form (1.2) then the unique solution (Y, Z, N) of the BSDE is reduced to (Y, Z) , *i.e.* the orthogonal martingale N is equal to zero showing that in a Markovian setting the "usual" solution (Y, Z) has not to be completed by a strongly orthogonal even if M does not enjoy the martingale representation property.

Sur la composante orthogonale d'une EDSR dans un contexte markovien

Résumé - Dans cette Note nous considérons une équation différentielle stochastique rétrograde (EDSR) de générateur déterministe et quadratique dirigée par une martingale continue M . Nous prouvons (dans le Théorème 2.1) que si M est un processus de Markov homogène fort et si l'EDSR est de la forme (1.2) l'unique solution (Y, Z, N) de l'EDSR se réduit à (Y, Z) , *i.e.* la martingale orthogonale N vaut zéro. Cela prouve que dans un contexte markovien la solution "habituelle" (Y, Z) n'a pas à être complétée par une martingale fortement orthogonale même si M ne possède pas la propriété de représentation martingale.

Version française abrégée

Dans cette Note nous considérons une équation différentielle stochastique rétrograde (EDSR) dirigée par une martingale continue M , de générateur quadratique f et admettant $F(X_T)$ pour condition terminale où $F : \mathbb{R} \rightarrow \mathbb{R}$ dénote une fonction déterministe suffisamment régulière et X l'unique solution forte d'une équation différentielle stochastique (EDS) également dirigée par M . Dans ce contexte il a été démontré dans [3] and [4] qu'il existe un unique triplet (Y, Z, N) solution de l'EDSR considérée où Y est un processus stochastique borné, Z un processus prévisible de carré intégrable et N une martingale fortement orthogonale à M . Puisque nous ne supposons pas que M possède la propriété de représentation martingale, la solution habituelle (Y, Z) doit *a priori* être complétée par une martingale N fortement orthogonale à M . Si le générateur f est supposé Lipschitz, les auteurs de [3] obtiennent la solution de l'EDSR (1.2) *via* une itération de Picard de la forme (2.1). Notons que la troisième composante de la solution, la martingale orthogonale N est "statique" lors de cette itération.

L'objet de cette Note est de démontrer que dans un contexte markovien (*i.e.* avec une condition terminale comme exposée plus haut et un générateur déterministe dépendant uniquement de y et z) la solution (Y, Z, N) se réduit au couple (Y, Z) autrement dit, la composante orthogonale N est nulle même si la propriété de représentation martingale n'est

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pas vérifiée pour M . Afin de simplifier la preuve du résultat principal (Théorème 2.1) nous considérons une diffusion X de dérive nulle et toutes les équations mises en jeu sont uni-dimensionnelles (le cas d'un générateur dépendant de (X, M) fera l'objet d'un travail futur). Ce résultat permettra (dans un travail en préparation) de simplifier l'étude des propriétés des EDSR quadratiques de la forme (1.2) comme en particulier donner une preuve de différentiabilité par rapport aux paramètres initiaux (x, m) (voir (1.1)) sans l'hypothèse additionnelle (MRP) (c.f. [2, Section 4.2]) utilisée dans [2, Theorem 4.6].

1 Preliminaries

Let $M := (M_t)_{t \in [0, T]}$ be a real-valued continuous square integrable martingale with respect to a continuous filtration $(\mathcal{F}_t)_{t \in [0, T]}$ both defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Assume that M is an homogeneous strong Markov process with respect to $(\mathcal{F}_t)_{t \in [0, T]}$. For (t, m) in $[0, T] \times \mathbb{R}$ we denote by $M^{t, m}$ the process defined as $M_s^{t, m} := m + M_s - M_t$, $s \in [t, T]$. Let $C := (C_t)_{t \in [0, T]}$ be the $(\mathcal{F}_t)_{t \in [0, T]}$ -predictable, increasing process defined by $C_t := \arctan(\langle M, M \rangle_t)$, $t \in [0, T]$. On this filtered probability space we also consider a stochastic process $X^{t, x, m} := (X_s^{t, x, m})_{s \in [t, T]}$ defined as the unique strong solution of the following one-dimensional stochastic differential equation

$$X_s^{t, x, m} = x + \int_t^s \sigma(X_r^{t, x, m}, M_r^{t, m}) dM_r, \quad s \in [t, T], \quad t \in [0, T] \quad (1.1)$$

where $\sigma : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ is deterministic, of class $C^2(\mathbb{R} \times \mathbb{R})$ with locally Lipschitz partial derivatives and such that there exists a positive constant k satisfying $|\sigma(x_1, m_1) - \sigma(x_2, m_2)| \leq k|x_1 - x_2|$, $\forall (x_1, x_2, m_1, m_2) \in \mathbb{R}^4$. Let us finally introduce the object of interest of this Note that is the following backward stochastic differential equation (BSDE) coupled with the forward process $X^{t, x, m}$ as

$$\begin{aligned} Y_s^{t, x, m} = & F(X_T^{t, x, m}) - \int_t^T Z_r^{t, x, m} dM_r + \int_t^T f(r, Y_r^{t, x, m}, Z_r^{t, x, m}) dC_r - \int_t^T dN_r^{t, x, m} \\ & + \frac{\kappa}{2} \int_t^T d\langle N^{t, x, m}, N^{t, x, m} \rangle_r. \end{aligned} \quad (1.2)$$

where $F : \mathbb{R} \rightarrow \mathbb{R}$ is a bounded deterministic function of class $C^2(\mathbb{R})$ with bounded derivatives. The generator $f : [0, T] \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ is assumed to be $\mathcal{B}([0, T]) \otimes \mathcal{B}(\mathbb{R}) \otimes \mathcal{B}(\mathbb{R})$ -measurable where $\mathcal{B}(\mathbb{R})$ is for the Borel σ -field on \mathbb{R} (so that $f(r, x, m)$ is deterministic for non-random (r, x, m) in $[0, T] \times \mathbb{R}^2$) and is such that there exists a deterministic constant c satisfying $\sup_{r \in [0, T]} |f(r, 0, 0)| \leq c$. We assume in addition that the generator f is quadratic in z and Lipschitz in y . The typical example being when f is of the form $f(s, y, z) = l(s, y) + \eta|z|^2$ where η is a fixed constant and l is Lipschitz in y (the more general "quadratic" assumptions can be found for example in [2]). We recall that in this setting, it is shown in [4] that there exists a unique triple $(Y^{t, x, m}, Z^{t, x, m}, N^{t, x, m}) \in \mathcal{S}^\infty \times L^2(d\langle M, M \rangle \otimes d\mathbb{P}) \times \mathcal{M}^2$ where \mathcal{S}^∞ is the space of bounded and continuous $(\mathcal{F}_t)_t$ -adapted processes, $L^2(d\langle M, M \rangle \otimes d\mathbb{P})$ denotes the space of square integrable $(\mathcal{F}_t)_t$ -predictable processes and \mathcal{M}^2 the space of square

integrable $(\mathcal{F}_t)_t$ -martingales N strongly orthogonal to M (i.e. $\langle M, N \rangle = 0$). We also mention that these processes are real-valued. We finally stress that all the conditions and assumptions previously mentioned will be assumed to hold in the rest of this Note and that K denotes a constant which can differ from one line to another. We conclude this section by recalling some important facts. First let us mention that only the couple (X, M) is an homogeneous strong Markov process.

Theorem 1.1. (*[1, Theorem (8.11)] or [5, V. Theorem 35]*) *The process $(X_s^{t,x,m}, M_s^{t,m})_{s \in [t,T]}$ is an homogeneous strong Markov process for the filtration $(\mathcal{F}_t)_{t \in [0,T]}$. If in addition M is assumed to enjoy the independent increments property then the stochastic process $(X_s^{t,x,m})_{s \in [t,T]}$ is a strong Markov process.*

The Markov property of the couple (X, M) transfers to the solution of (1.2) and (2.2).

Theorem 1.2. (*[2, Proposition 3.2, Theorem 3.4]*) *There exist two deterministic functions $u, v : [0, T] \times \mathbb{R}^2 \rightarrow \mathbb{R}$, $\mathcal{B}([0, T]) \otimes \mathcal{B}(\mathbb{R}^2)$ such that $(Y^{t,x,m}, Z^{t,x,m})$ in (1.2) and (2.2) satisfy*

$$Y_s^{t,x,m} = u(s, X_s^{t,x,m}, M_s^{t,m}), \quad Z_s^{t,x,m} = v(s, X_s^{t,x,m}, M_s^{t,m})\sigma(s, X_s^{t,x,m}, M_s^{t,m}), \quad s \in [t, T]$$

where $\mathcal{B}_e(\mathbb{R}^2)$ is the σ -field on \mathbb{R}^2 generated by functions $(x, m) \mapsto \mathbb{E} \left[\phi(s, X_s^{t,x,m}, M_s^{t,m}) dC_s \right]$ with $\phi : \Omega \times [0, T] \times \mathbb{R}^2 \rightarrow \mathbb{R}$ a continuous bounded function.

Finally we will use the following ²property for the solution of the BSDE (2.2).

Theorem 1.3. (*Particular case of [2, Theorem 4.6]*) *The map $(x, m) \mapsto Y^{1,t,x,m}$ is of class $\mathcal{C}^1(\mathbb{R}^2)$ \mathbb{P} -a.s. where $Y^{1,t,x,m}$ is as in (2.2) below.*

2 Main result

We are now ready to state and prove the main result of this Note.

Theorem 2.1. *Assume that assumptions of Section 1 are in force then $N^{t,x,m}$ in (1.2) is equal to zero and equation (1.2) becomes*

$$Y_s^{t,x,m} = F(X_T^{t,x,m}) - \int_t^T Z_r^{t,x,m} dM_r + \int_t^T f(r, Y_r^{t,x,m}, Z_r^{t,x,m}) dC_r.$$

Proof. First note that it is enough to assume that the generator f is Lipschitz in (y, z) . Indeed, in [4, Theorems 2.5-2.6], the existence and uniqueness of the solution $(Y^{t,x,m}, Z^{t,x,m}, N^{t,x,m})$ of the BSDE (1.2) is given as a limit of solutions of Lipschitz BSDEs. As a consequence, $N^{t,x,m}$ is equal to zero in (1.2) if the orthogonal martingale parts N in the approximating

²Note that this result ([2, Theorem 4.6]) has been proved under an additional technical assumption (MRP) with f a quadratic generator. Since the generator in equation (2.2) is very simple, using only an exponential change we can apply the computations realized in [2, Theorem 4.6] without assuming the hypothesis (MRP). The full proof of this fact will be presented in a paper in preparation.

Lipschitz BSDEs vanish. So assume f to be Lipschitz in (y, z) . In [3] the authors show that the unique solution of (1.2) is obtained as the limit of the following Picard iteration:

$$\begin{aligned} Y_s^{0,t,x,m} &= Z_s^{0,t,x,m} = 0, \\ Y_s^{k+1,t,x,m} &= F(X_T^{t,x,m}) - \int_s^T Z_r^{k,t,x,m} dM_r + \int_s^T f(r, Y_r^{k,t,x,m}, Z_r^{k,t,x,m}) dC_r \\ &\quad - \int_s^T dN_r^{t,x,m} + \kappa \int_s^T d\langle N^{t,x,m}, N^{t,x,m} \rangle_r, \quad k \geq 0. \end{aligned} \quad (2.1)$$

Note that $N^{t,x,m}$ is not part of the iteration (we refer to [3, Proof of Theorem 6.1] for more details). This remark leads to the main idea of the proof. Indeed, consider the first iteration, *i.e.* $(Y^{1,t,x,m}, Z^{1,t,x,m}, N^{t,x,m})$ defined by

$$Y_s^{1,t,x,m} = F(X_T^{t,x,m}) - \int_s^T Z_r^{1,t,x,m} dM_r + \int_s^T f(r, 0, 0) dC_r - \int_s^T dN_r^{t,x,m} + \kappa \int_s^T d\langle N^{t,x,m}, N^{t,x,m} \rangle_r. \quad (2.2)$$

By the a priori estimates obtained in [3, Proposition 6.3] the triplet $(Y^{1,t,x,m}, Z^{1,t,x,m}, N^{t,x,m})$ is unique. As a consequence if we show that $N^{t,x,m} = 0$ in equation (2.2) then the Theorem is proved since $(Y^{k,t,x,m}, Z^{k,t,x,m}, N^{t,x,m})$ converges to the unique solution of (1.2). The rest of the proof is devoted to this fact.

Since $Y^{1,t,x,m}$ is (\mathcal{F}) -adapted it holds by Markov property that

$$Y_s^{1,t,x,m} = g(s, X_s^{t,x,m}, M_s^{t,x,m}), \quad \text{with } g(s, x, m) := \mathbb{E} \left[F(X_{T-s}^{t,x,m}) - \int_s^T f(r, 0, 0) dC_r \right].$$

In addition, Proposition 1.3 applied to (2.2) gives that the application $(x, m) \mapsto g(t, x, m)$ is of class $\mathcal{C}^1(\mathbb{R} \times \mathbb{R})$ for every t . We mimic a technique given in [2] and compute $\langle Y^{1,t,x,m}, N^{1,t,x,m} \rangle_s$ for $s \geq t$. Let $\pi^{(n)} := \{t = t_0^{(n)} \leq t_1^{(n)} \leq \dots \leq t_N^{(n)} = s\}$ be a family of subdivisions of $[t, s]$ whose mesh $|\pi^{(n)}|$ tends to zero as n goes to the infinity. For sake of simplicity the superscript (n) will be omitted in the following computations.

$$\begin{aligned} \langle Y^{1,t,x,m}, N^{t,x,m} \rangle_s &= \langle g(\cdot, X^{t,x,m}, M^{t,x,m}), N^{t,x,m} \rangle_s \\ &\stackrel{\mathbb{P}}{=} \lim_{n \rightarrow \infty} \sum_{j=1}^r (g(t_{j+1}, X_{t_{j+1}}^{t,x,m}, M_{t_{j+1}}^{t,x,m}) - g(t_j, X_{t_j}^{t,x,m}, M_{t_j}^{t,x,m})) \Delta_j N^{t,x,m} \\ &\stackrel{\mathbb{P}}{=} \lim_{n \rightarrow \infty} \sum_{j=1}^r \left[(g(t_j, X_{t_{j+1}}^{t,x,m}, M_{t_{j+1}}^{t,x,m}) - g(t_j, X_{t_j}^{t,x,m}, M_{t_j}^{t,x,m})) \Delta_j N^{t,x,m} \right. \\ &\quad \left. + (g(t_{j+1}, X_{t_{j+1}}^{t,x,m}, M_{t_{j+1}}^{t,x,m}) - g(t_j, X_{t_{j+1}}^{t,x,m}, M_{t_{j+1}}^{t,x,m})) \Delta_j N^{t,x,m} \right]. \end{aligned} \quad (2.3)$$

We consider the two sumands above separately. For the first part we follow a technique used in [2] and apply the mean theorem. Let \bar{M}_j (respectively \bar{X}_j) below a random point between $M_{t_j}^{t,x,m}$ and $M_{t_{j+1}}^{t,x,m}$ (resp. $X_{t_j}^{t,x,m}$ and $X_{t_{j+1}}^{t,x,m}$) in the computations below. We have

$$\sum_{j=1}^r (g(t_j, X_{t_{j+1}}^{t,x,m}, M_{t_{j+1}}^{t,x,m}) - g(t_j, X_{t_j}^{t,x,m}, M_{t_j}^{t,x,m})) \Delta_j N^{t,x,m}$$

$$\begin{aligned}
&= \sum_{j=1}^r (g(t_j, X_{t_{j+1}}^{t,x,m}, M_{t_{j+1}}^{t,x,m}) - g(t_j, X_{t_j}^{t,x,m}, M_{t_{j+1}}^{t,x,m})) \Delta_j N^{t,x,m} \\
&\quad + \sum_{j=1}^r (g(t_j, X_{t_j}^{t,x,m}, M_{t_{j+1}}^{t,x,m}) - g(t_j, X_{t_j}^{t,x,m}, M_{t_j}^{t,x,m})) \Delta_j N^{t,x,m} \\
&= \sum_{j=1}^r \left[\partial_2 g(t_j, X_{t_j}^{t,x,m}, M_{t_j}^{t,x,m}) \Delta_j X \Delta_j N^{t,x,m} + \partial_3 g(t_j, X_{t_j}^{t,x,m}, M_{t_j}^{t,x,m}) \Delta_j M \Delta_j N^{t,x,m} + R_{j,r} \right] \quad (2.4)
\end{aligned}$$

where $R_{j,r}$ is defined as

$$\begin{aligned}
R_{j,r} &:= (\partial_2 g(t_j, \bar{X}_j, M_{t_{j+1}}^{t,x,m}) - \partial_2 g(t_j, X_{t_j}^{t,x,m}, M_{t_j}^{t,x,m})) \Delta_j X \Delta_j N^{t,x,m} \\
&\quad + (\partial_3 g(t_j, X_{t_j}^{t,x,m}, \bar{M}_j) - \partial_3 g(t_j, X_{t_j}^{t,x,m}, M_{t_j}^{t,x,m})) \Delta_j M \Delta_j N^{t,x,m}.
\end{aligned}$$

Since $(x, m) \mapsto g(s, x, m)$ is of class \mathcal{C}^1 for every s in $[0, T]$ the remainder term $\sum_{j=0}^r R_{j,r}$ as r goes to infinity (we refer to [2, Proof of (5.13)] for the complete justifications). Then it follows using (2.4) that

$$\begin{aligned}
&\lim_{r \rightarrow \infty} \sum_{j=1}^r (g(t_j, X_{t_{j+1}}^{t,x,m}, M_{t_{j+1}}^{t,x,m}) - g(t_j, X_{t_j}^{t,x,m}, M_{t_j}^{t,x,m})) \Delta_j N^{t,x,m} \\
&= \langle \int_t^\cdot \partial_2 g(r, X_r^{t,x,m}, M_r^{t,x,m}) \sigma(r, X_r^{t,x,m}, M_r^{t,x,m}) + \partial_3 g(r, X_r^{t,x,m}, M_r^{t,x,m}) dM_r, N^{t,x,m} \rangle_s = 0
\end{aligned}$$

by strong orthogonality between M and N . As a consequence, relation (2.3) reduces to

$$\langle Y^{1,t,x,m}, N^{t,x,m} \rangle_s \stackrel{\mathbb{P}}{=} \lim_{n \rightarrow \infty} \sum_{j=1}^r (g(t_{j+1}, X_{t_{j+1}}^{t,x,m}, M_{t_{j+1}}^{t,x,m}) - g(t_j, X_{t_{j+1}}^{t,x,m}, M_{t_{j+1}}^{t,x,m})) \Delta_j N^{t,x,m}. \quad (2.5)$$

We have that

$$\begin{aligned}
&\left| \sum_{j=1}^r (g(t_{j+1}, X_{t_{j+1}}^{t,x,m}, M_{t_{j+1}}^{t,x,m}) - g(t_j, X_{t_{j+1}}^{t,x,m}, M_{t_{j+1}}^{t,x,m})) \Delta_j N^{t,x,m} \right|^2 \\
&\leq \sum_{j=1}^n \left| g(t_{j+1}, X_{t_{j+1}}^{t,x,m}, M_{t_{j+1}}^{t,x,m}) - g(t_j, X_{t_{j+1}}^{t,x,m}, M_{t_{j+1}}^{t,x,m}) \right|^2 \times \sum_{j=1}^n |\Delta_j N|^2 \\
&= \sum_{j=1}^n \left| \mathbb{E} \left[F(X_{T-t_{j+1}}^{0, X_{t_{j+1}}^{t,x,m}, M_{t_{j+1}}^{t,x,m}}) - F(X_{T-t_j}^{0, X_{t_{j+1}}^{t,x,m}, M_{t_{j+1}}^{t,x,m}}) - \int_{t_j}^{t_{j+1}} f(r, 0, 0) dC_r \right] \right|^2 \times \sum_{j=1}^n |\Delta_j N|^2 \\
&\leq 2 \left[\sum_{j=1}^n \left| \mathbb{E} [F(\tilde{X}_{T-t_{j+1}}) - F(\tilde{X}_{T-t_j})] \right|^2 + \sum_{j=1}^n \left| \mathbb{E} \left[\int_{t_j}^{t_{j+1}} f(r, 0, 0) dC_r \right] \right|^2 \right] \times \sum_{j=1}^n |\Delta_j N|^2
\end{aligned}$$

where for simplicity of notations we set $\tilde{X}_s := X_s^{0, X_{t_{j+1}}^{t,x,m}, M_{t_{j+1}}^{t,x,m}}$. Let \bar{X}_j be a random point between $\tilde{X}_{T-t_{j+1}}$ and \tilde{X}_{T-t_j} . Writing $\mathbb{E} [F(\tilde{X}_{T-t_{j+1}}) - F(\tilde{X}_{T-t_j})]$ as

$$\mathbb{E} [F(\tilde{X}_{T-t_{j+1}}) - F(\tilde{X}_{T-t_j})]$$

$$\begin{aligned}
&= \mathbb{E} \left[F'(\tilde{X}_{T-t_{j+1}}) (\tilde{X}_{T-t_{j+1}} - \tilde{X}_{T-t_j}) \right] + \frac{1}{2} \mathbb{E} \left[F''(\bar{X}_j) |\tilde{X}_{T-t_{j+1}} - \tilde{X}_{T-t_j}|^2 \right] \\
&= \mathbb{E} \left[F'(\tilde{X}_{T-t_{j+1}}) \mathbb{E} [\tilde{X}_{T-t_{j+1}} - \tilde{X}_{T-t_j} | \mathcal{F}_{T-t_{j+1}}] \right] + \frac{1}{2} \mathbb{E} \left[F''(\bar{X}_j) |\tilde{X}_{T-t_{j+1}} - \tilde{X}_{T-t_j}|^2 \right] \\
&\leq K \mathbb{E} \left[|\tilde{X}_{T-t_{j+1}} - \tilde{X}_{T-t_j}|^2 \right]
\end{aligned}$$

and $f(r, 0, 0)$ as $f(r, 0, 0) = \max\{f(r, 0, 0), 0\} - \max\{-f(r, 0, 0), 0\}$ it follows that

$$\begin{aligned}
&\left| \sum_{j=1}^n (g(t_{j+1}, M_{t_{j+1}}) - g(t_j, M_{t_j})) \Delta_j N \right|^2 \\
&\leq K \left[\sum_{j=1}^n \left| E [|\tilde{X}_{T-t_{j+1}} - \tilde{X}_{T-t_j}|^2] \right|^2 + \sup_{r \in [0, T]} |f(r, 0, 0)| \sum_{j=1}^n |\mathbb{E} [C_{t_{j+1}} - C_{t_j}]|^2 \right] \times \sum_{j=1}^n |\Delta_j N|^2 \\
&\leq K \left[\sum_{j=1}^n E [|\tilde{X}_{T-t_{j+1}} - \tilde{X}_{T-t_j}|^4] + \sup_{r \in [0, T]} |f(r, 0, 0)| \sum_{j=1}^n \mathbb{E} [|M_{t_{j+1}} - M_{t_j}|^4] \right] \times \sum_{j=1}^n |\Delta_j N|^2 \\
&\leq K \left(E \left[\sum_{j=1}^{\infty} |\tilde{X}_{T-t_{j+1}} - \tilde{X}_{T-t_j}|^4 \right] + \mathbb{E} \left[\sum_{j=1}^{\infty} |M_{t_{j+1}} - M_{t_j}|^4 \right] \right) \times \sum_{j=1}^{\infty} |\Delta_j N|^2 \\
&= 0
\end{aligned}$$

since the quartic variations of a martingale are zero. The previous computation and the equality (2.5) entail that

$$\langle Y^{1,t,x,m}, N^{t,x,m} \rangle_s \stackrel{\mathbb{P}}{=} 0. \quad (2.6)$$

On the other hand, the covariation $\langle Y^{1,t,x,m}, N^{t,x,m} \rangle_s$ in the BSDE (2.2) equals to

$$\langle Y^{1,t,x,m}, N^{t,x,m} \rangle_s \stackrel{\mathbb{P}}{=} \langle N^{t,x,m}, N^{t,x,m} \rangle_s. \quad (2.7)$$

Hence relations (2.6) and (2.7) give that $N_s^{t,x,m} = N_0^{t,x,m}$ for every s in $[t, T]$. \square

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